Informality, Heterogeneity, and the Business Cycle

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Abstract

This paper develops a quantitative framework to analyze how informal labor markets shape employment dynamics over the business cycle in developing economies. We construct a search and matching model with heterogeneous workers, endogenous firm entry in formal and informal sectors, and aggregate productivity shocks. The model's block recursive structure provides a tractable way to incorporate worker heterogeneity while dealing with aggregate uncertainty. In our numerical analysis, informal employment is an important margin of adjustment during economic downturns, particularly for low-skilled workers. The presence of an informal sector reduces unemployment and improves lifetime utility for low-skilled workers despite paying lower wages. Through counterfactual policy experiments, we examine how formal sector regulations impact sectoral choices and highlight the potential trade-offs between reducing informality and increasing unemployment over the business cycle. Our findings emphasize the importance of considering distributional effects and cyclical dynamics when designing labor market policies in economies with substantial informal employment.

Keywords: Informal Sector; Cyclical Unemployment; Labor Market Institutions, Labor Market Policies, Regulations

JEL Codes: E24; E26; J08

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1 Introduction

Labor market informality is a defining characteristic of developing economies, with informal employment accounting for a substantial share of total employment. While often viewed as a symptom of institutional and regulatory failures, the informal sector plays a complex dual role. Although avoiding regulations undermines government tax revenue and compromises worker rights (as informality implies evading direct costs, such as taxes and social security contributions), it also means bypassing the operational rigidities (like hiring and firing rules) that can limit flexibility to changing economic conditions.

The state of the business cycle can crucially impact the decision process of firms and workers regarding informality, as both firms' incentives to create formal jobs and workers' valuation of informal employment opportunities vary with the aggregate conditions of the economy. This feature, however, is often overlooked in the literature. Despite the empirical evidence on the counter-cyclical behavior of informality and the differences in the patterns of the flows to and from unemployment of workers with distinct formality statuses, much of the previous work has focused on steady-state economies with no space for changing aggregate productivity.

This paper develops a quantitative framework to analyze how the presence of an informal sector shapes labor market dynamics over the business cycle. The model allows us to focus on two aspects: (1) the aggregate implications of informality for labor market outcomes during economic fluctuations, and (2) the distributional effects across workers with different skill levels.

Our framework displays heterogeneous workers who differ in their human capital and firms that can choose between creating formal or informal jobs. Hence, we focus on informality as informal wage work hired by firms rather than a frictionless self-employment state of the workers. Aggregate productivity shocks generate business cycle fluctuations, endogenous separations, and changes in the optimal formality sector choice of workers and firms.

Our methodological contribution lies in developing a tractable framework while incorporating worker heterogeneity, search and matching frictions, and aggregate uncertainty. The key to achieving this is the model's block recursive structure (Menzio and Shi, 2010, 2011; Kaas, 2023). This feature ensures that agents' value and policy functions depend on the aggregate state only through the level of aggregate productivity, eliminating the dependency on the infinite-dimensional distribution of workers across states. Block recursivity emerges as a combination of (i) directed search, in the sense that firms post vacancies in specific submarkets and workers self-select into their preferred locations, (ii) free entry of firms, and (iii) a constant returns to scale matching function. These three elements allow the market tightness of each submarket to be determined independently of the workers' distribution, and it thus compresses all information necessary for the worker's choice. This contrasts with random search models, where firms and workers face uncertainty about their potential matches, yielding value functions dependent on the full distribution of worker types and states.

We calibrate the model to match the key qualitative patterns of labor markets in Brazil, and then simulate it to illustrate the dynamic behavior of informality and employment through aggregate fluctuations and policy counterfactuals. We demonstrate that informal employment can serve as a crucial margin of adjustment during economic downturns, particularly for low-skilled workers. In the baseline economy, the presence of an informal sector reduces unemployment and improves welfare for low-skilled workers, even though it has lower wages. High-skilled workers, operating almost exclusively in the formal sector, face similar conditions in the alternative scenario of the absence of informality. These findings mark the importance of the distributional effects of the cyclical dynamic of formal and informal jobs.

In counterfactual policy experiments, we examine the impact of formal sector regulations. Not only do these margins impact the sectoral decisions of workers and firms, but they are also some of the feasible instruments in efforts to tame down informality. Hence, it is crucial for policymakers to know how efficient each particular intervention is, its aggregate impacts, its distributional consequences, and the relevance of the timing of the policy action along the business cycle. The potential trade-offs between less informality and more unemployment, particularly during downturns, add to the complexity of designing labor market policies in economies with substantial informal employment.

We build on a rich body of work analyzing labor market informality in steady-state environments. This literature has enhanced our understanding of the incentives faced by firms and workers and their implications on firm dynamics, productivity, and earnings, while also assessing the impacts of various policy and regulatory changes (Haanwinckel and Soares, 2021; Machado Parente, Brotherhood, and Iachan, 2025; Meghir, Narita, and Robin, 2015; Ulyssea, 2018, 2020). However, they abstract from the role of business cycle fluctuations.

The cyclical properties of informal labor markets are the focus of a relatively smaller literature. Shapiro (2014) studies the impact of having a large self-employed "sector" on the behavior of employment and output along the business cycle. Fiess, Fugazza, and Maloney (2010) also treat the informal sector as self-employment to understand its impact on the transmission of economic shocks. Fernández and Meza (2015), Leyva and Urrutia (2020), and Horvath and Yang (2022) build business cycle models with informal sectors and measure the impact of informality and labor regulation on macroeconomic volatility and the propagation of shocks. The first two also model informality as a frictionless self-employment state. Thus, they abstract from the intensive margin of informality, through which formal firms hire workers off the books. Closer to our framework is Bosch and Esteban-Pretel (2012). However, as in the previously mentioned papers, they model workers as ex-ante homogeneous. The block recursive setting of our framework allows us to study the distributional effects of the interaction of the business cycle and informality, as well as the impact of policy changes, by incorporating worker heterogeneity. We build on the search-and-matching tradition (e.g., Shimer (2005)), where aggregate productivity shocks interact with search frictions to drive labor market dynamics. On the firm's side, we endogenize both the job-finding and the separation rates, as well as the intensive margin of informality, by allowing firms to optimally choose when to create and terminate vacancies in both sectors.

In what follows, we start presenting a brief empirical overview of the cyclical behavior of labor markets in Brazil (Section 2). Section 3 describes our model, while Section 4 presents the parametrization and the results of our quantitative exercises. Section 5 concludes with the final remarks.

2 Empirical Evidence

In this section, we provide empirical evidence on the cyclical patterns of informality in developing economies using Brazil as our setting. We document three empirical regularities that motivate the key features of our model: the counter-cyclical nature of informality, heterogeneity in labor market outcomes across skill groups, and wage differences across sectors.

We use microdata from *Pesquisa Mensal de Emprego Nova* (PME), a rotating household survey with monthly visits that covered urban areas of six large metropolitan regions. Its data is available for the period from 2002.03 to 2016.02.[1] We restrict our sample to prime-age workers (18-59 years). We adopt the methodology implemented by Data Zoom (2025) to explore the panel aspect of the microdata and construct indicators of the labor market transitions experienced by each worker over subsequent interviews.

Besides the fact that informality is a widespread phenomenon in the country, Brazil is an adequate setting for studying the topic because its institutional design provides a clear and widely understood notion of what constitutes a formal job: labor regulation requires that, when formally hiring a worker, the firm sign a document called *Carteira de Trabalho e Previdência Social* (CTPS). By doing so, the firm informs competent authorities of the hiring, committing itself to paying all mandatory benefits to the worker (such as compliance with the minimum wage and contributions to social security and a fund available upon layoff) and associated payroll taxes. Workers are aware of whether they were hired under a signed CTPS, and the survey design explicitly asks employed individuals if their work regime follows this formalized scheme or not. Thus, this provides a clear and effective way to classify the workers in the sample into the formal or the informal sector.

Workers in the sample belong to one out of seven employment statuses. The model we present in the next section includes three: employed in the private formal sector (those with a CTPS contract), employed as an informal wage worker (those working for someone else without a CTPS contract), and unemployed. The remaining statuses, excluded from the model, are self-employed, employers, public sector/military workers, and those out of the labor force.

Table 1 presents the cyclical properties of key labor market stocks (in logs). It reports the standard deviation and the correlation with economic activity of employment rates as a share of the working-age population, the unemployment rate among the private sector labor force, and

[1] We acknowledge two primary limitations of using PME: its discontinuation in 2016 and its restricted coverage to six metropolitan regions. Despite the availability of the more recent, nationally representative *Pesquisa Nacional por Amostra de Domicílios Contínua* (PNADC), we find PME better suited to our focus on business cycle fluctuations. PME offers a higher frequency than PNADC (monthly vs quarterly), has a longer temporal span, and covers three distinct recessionary periods (2003, 2008-2009, and 2014-2016). Conversely, PNADC only captures the 2014-2016 crisis and the COVID-19 recession, with the latter having significant analytical complications due to data collection disruptions and its highly atypical nature, making it difficult to compare with standard business cycles. We confirm, however, that the primary patterns documented here are also present in the PNADC data, and these results are available upon request.

the informality rate as a share of total private employment.[2] All series are seasonally adjusted using the X-13 ARIMA-SEATS method and further smoothed using a centered moving average of adjacent periods to minimize high-frequency noise.

The data is in line with previous findings of the literature on counter-cyclical behavior of the informality rate (Bosch & Esteban-Pretel, 2012; Bosch & Maloney, 2008; Loayza & Rigolini, 2011; Roldos et al., 2019). While total and formal employment show pro-cyclical behavior, informal employment's cyclicality has the opposite sign, suggesting its role as a buffer during economic fluctuations. Notably, in our sample, this counter-cyclical behavior seems to be primarily driven by the informal wage work rather than self-employment. In terms of variance over time, formal employment shows a somewhat more stable trajectory than informal wage work. These patterns are consistent across skill groups: both low and high-skilled workers experience counter-cyclical informality rates and pro-cyclical formal employment, with similar magnitudes (see Table 2). The evidence suggests that the aggregate patterns thus reflect a systematic feature of the labor market rather than compositional changes.

Table 1: Labor Market Stocks

Variable	S.D.	Corr. with activity
Unemployment (% private labor force)	0.299	-0.427
Informality (% private employed)	0.221	-0.301
Employment (% work. age pop.)	0.037	0.360
Formal private employment (% work. age pop.)	0.123	0.313
Informal wage employment (% work. age pop.)	0.178	-0.275
Self-employment (% work. age pop.)	0.024	-0.099

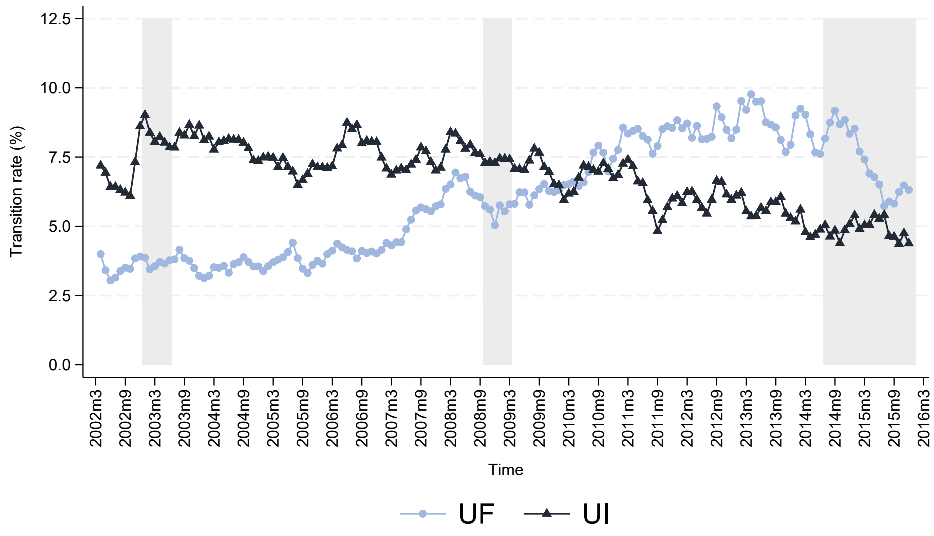
Notes: logs of seasonally adjusted stocks by the X-13 ARIMA-SEATS method and further smoothed by a centered-moving average using the two adjacent periods. Stocks calculated using the surveys' expansion weights. Monthly activity measure is the cycle component of the smoothed and seasonally adjusted log of the IBC-BR index from Banco Central do Brasil (2025).

We now turn to the analysis of labor market flows. Figure 1 presents monthly transition rates between employment states over the 2002-2016 horizon of PME, a period that provides a comprehensive view of labor market dynamics across three recession and expansion cycles. Panel (a) shows job-finding rates from unemployment into formal employment (UF) and into informal wage employment (UI). Particularly during the 2014-2016 recession, the formal sector job-finding rate exhibits a substantial decline, while transitions into informal jobs remain relatively stable.

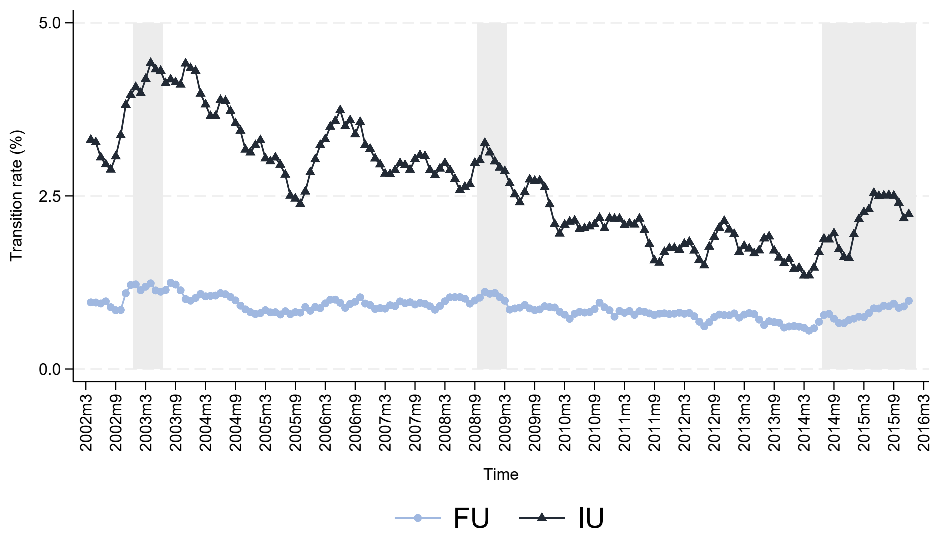
[2]We treat the private sector as the combination of formal private workers and informal wage workers. The private labor force adds the unemployed to this last group. These definitions are consistent with the employment conditions of our model.

Figure 1: Cyclicality of labor market flows

(a) Job-finding rate



(b) Job-separation rate



Notes: monthly flows from PME, seasonally adjusted by the X-13 ARIMA-SEATS method and further smoothed by a centered-moving average using the two adjacent periods. Flows calculated using the survey's expansion weights. Shaded areas indicate recession periods as defined by CODACE (2023). "U" stands for unemployment, "F" for formal employment, "I" for informal wage work, and "S" for self-employment.

Panel (b), which displays job-separation rates (FU, IU), reveals important differences in job stability. The formal sector is characterized by consistently lower and more stable separation rates, averaging just 0.88% per month with only modest increases during downturns. The informal sector, in contrast, exhibits much higher instability: its separation rate (IU) averages 2.65% per month (nearly three times the formal rate) and rises significantly during recessions. This heightened cyclicality of the informal separation rate aligns with the findings of Gomes, Iachan, and Santos (2020), who also document that flows from informal employment to unemployment are more pronounced during recessions – a finding they obtain using the PNADC survey over a more recent period. This difference in cyclicality is likely a result of the firing costs imposed by formal sector regulations, which are absent in informal work. We capture this distinction in our model through sector-specific firing costs and exogenous separation rates.

These patterns suggest that the counter-cyclical nature of informality stems primarily from changes in job creation rather than job destruction. During economic downturns, while formal job opportunities become scarce, the informal sector continues to absorb workers despite its higher turnover rates. This pattern highlights the potential role of informal employment as a buffer against unemployment during adverse economic conditions, providing an alternative source of income when formal jobs are harder to secure.

Table 2 summarizes key labor market outcomes by skill level, defined by high school completion (high-skilled) or non-completion (low-skilled). We use this criterion to map our model's skill groups to the data. Unemployment rates exhibit similar average levels, dispersion, and cyclical behavior across both worker types. This similarity also extends to separation rates.

Table 2: Labor Market Outcomes by Skill Level

Variable	Mean	S.D.	Corr. with activity
Low-skilled workers			
Unemployment rate	11.67	3.62	-0.367
Informality rate	33.88	4.65	-0.325
Job-finding: formal (UF)	5.38	2.07	0.379
Job-finding: informal (UI)	8.34	1.10	-0.214
Separation: formal (FU)	0.85	0.14	-0.367
Separation: informal (IU)	2.62	0.81	-0.361
High-skilled workers			
Unemployment rate	11.45	3.14	-0.405
Informality rate	19.04	4.11	-0.282
Job-finding: formal (UF)	6.52	1.94	0.424
Job-finding: informal (UI)	5.62	1.05	-0.252
Separation: formal (FU)	0.90	0.16	-0.413
Separation: informal (IU)	2.70	0.83	-0.348

Notes: Low-skilled = less than high school completion; High-skilled = high school degree or more. Rates in percentages. All series seasonally adjusted using X-13 ARIMA-SEATS method and further smoothed by a centered-moving average using the two adjacent periods. Monthly activity measure is the cycle component of the smoothed and seasonally adjusted log of the IBC-BR index.

In contrast, informality rates are markedly higher among low-skilled workers, averaging almost 34% in the period, versus only 19% among high-skilled workers. The job-finding rates reveal the source of this difference: high-skilled workers are more likely to find a formal job, while low-skilled workers face a higher probability of finding informal employment than formal employment. Importantly, the cyclical properties are qualitatively similar across skill groups—both experience counter-cyclical unemployment and informality. This suggests that aggregate shocks affect the entire labor market, albeit with potentially different intensities.

While the aggregate stocks and flows define the market's dynamics, the wage structure reveals the underlying incentives for workers. Table 3 presents key statistics on the wage structure by sector and skill. Panel A reports average real hourly wages. Unsurprisingly, wages are higher for high-skilled workers and for those employed in the formal sector.

Table 3: Wage Structure

Panel A: Average hourly wages		
	Low-Skilled	High-Skilled
Formal sector	10.45	20.84
Informal sector	8.39	16.67
Panel B: Sector and skill premiums		
Formal premium (log points)	0.0751	
High-skill premium (log points)	0.5878	

Notes: Panel A: wages in BRL from December 2024. Panel B: Formal premium from worker fixed effects regression controlling for age and age squared. High-skill premium from pooled OLS controlling for age, age squared, gender, and metropolitan area fixed effects. See full regression results on Table B3 in Appendix B.

However, simple wage averages are insufficient as they are skewed by significant selection effects — for example, higher-ability workers may disproportionately sort into formal jobs. In Panel B, we show our estimations of the sector and skill wage premiums.

To isolate the formal wage premium from time-invariant worker characteristics (including unobserved factors such as innate ability or motivation), we use an individual fixed-effects (FE) model on the panel of log real hourly wages. The specification also includes controls for age and its square:

$$\ln(w_{it}) = \alpha_i + \beta_{formal}\text{Formal}_{it} + \gamma_1 \text{age}_{it} + \gamma_2 \text{age}_{it}^2 + \epsilon_{it}$$

The coefficient of interest, β_{formal}, captures the wage change a worker experiences when switching sectors, purged of their fixed personal attributes. We find a positive coefficient of 0.0751, which implies that, after controlling for worker-level characteristics, formal sector wages are 7.8% higher on average than informal wages.[3]

The fixed-effects model, by design, cannot estimate the premium for a time-invariant characteristic like skill. Therefore, to identify the skill wage premium, we estimate a separate

[3]The percentage is calculated as $(\exp(\hat{\beta}) - 1) \times 100$.

pooled OLS Mincer regression of log real hourly wages on the high-skill dummy. This specification allows us to quantify the wage gap between skill groups while controlling for a set of observable characteristics:[4]

$$\ln(w_i) = \alpha + \beta_{skill}\text{HighSkill}_i + \mathbf{\Gamma} X_i + \epsilon_i.$$

The resulting estimate of coefficient β_{skill} is 0.5878, indicating that high-skilled workers earn, on average, 80% more than their low-skilled counterparts after controlling for formality, age, and demographics. Together, these patterns motivate our model's specification of higher productivity in the formal sector and skill-differentiated human capital.

3 The Model

In this section, we present a search and matching model of a labor market with formal and informal sectors. The model is designed to capture the key empirical patterns documented in the previous section, particularly the cyclical dynamics between formal and informal employment and the heterogeneity across skill levels. The model features directed search by unemployed workers and free entry by firms, which endogenously determine the market tightness for all potential submarkets.

The economy is populated by a continuum of workers of two skill types $h \in \{L = \text{low}, H = \text{high}\}$, with measures $\pi_L + \pi_H = 1$. There is a continuum of firms with positive measure. Each worker has an endowment of 1 (indivisible) unit of labor. Workers have linear instantaneous utility and all agents have discount factor β.[5]

There are two types of job relations ("sectors"), $s \in \{F, I\}$, where F stands for formal and I stands for informal. A job in the formal sector is subject to a minimum wage, a benefit for the worker in case of losing the job, and the application of costs: (i) wage taxes for the workers; (ii) payroll and profit taxes for the firms; and (iii) firing costs for the firms. An informal job is not subject to any of these labor market regulations. However, it is subject to a higher exogenous separation rate and uses a less productive technology.

There is an aggregate productivity shock z. The productivity of a job depends on the skill level h of the worker and has an idiosyncratic time-fixed component, y, which is drawn after the match occurs from a distribution G. The production technology is sector-dependent, with $f(h, s, y; z) = \exp\{A_s + z + y\} \times h$, where $A_F > A_I$ captures the fact that formal firms have access to better institutions, such as credit markets, litigation services, etc. The vacancy posting cost in sector s is κ_s. Unemployed workers have home production $b(h)$.

Each worker searching for a job chooses a particular submarket indexed by $\eta = (h, s, w; z)$, where h is their human capital level, s is the sector, w is the wage offered, and z is the aggregate state of the economy. Note that, as skill level h and the aggregate state z are given, the choice

[4]We include age and its square, gender, region, and a formal sector indicator.

[5]Linear utility allows for a simple way to model unemployment insurance without introducing state-dependent utility functions.

component of a submarket is (s, w). There is a constant return to scale matching function $M(u, v)$, where u is the number of unemployed workers and v is the number of vacancies. Each submarket has market tightness $\theta(\eta) = \frac{v(\eta)}{u(\eta)} \geq 0$, which is determined in equilibrium. The probability of a worker in a given market being selected to a match and the probability of a firm filling a vacancy posted in a market are, respectively:

$$p\big(\theta(\eta)\big) = \frac{M\big(u(\eta), v(\eta)\big)}{u(\eta)} = M\big(1, \theta(\eta)\big) \tag{1}$$

$$q\big(\theta(\eta)\big) = \frac{p\big(\theta(\eta)\big)}{\theta(\eta)}. \tag{2}$$

For each period t, the timing of the economy is as follows:

i) The aggregate uncertainty is resolved.
ii) Separations happen exogenously and as an endogenous choice of the firms.
iii) Unemployed workers search, deciding which market to visit.
iv) Matches occur and productivity levels are drawn for the newly formed jobs.
v) Production, consumption, and tax payments occur.

3.1 Bellman Equations

Unemployed worker

An unemployed worker with skill level h enjoys utility from their home production level $b(h)$ and chooses the sector s and wage w that characterize a submarket $\eta = (h, s, w; z)$ to search for a job, given aggregate productivity level z. They are matched to a vacancy with probability $p(\theta(\eta))$. If the match happens, the productivity y is drawn from G and they become an employed worker with value function $W(h, s, w, y; z')$ next period. We denote by $m^U(h; z) \in (S \times W)$ the policy function of the optimal submarket choice of the unemployed worker. The value function for an unemployed worker is

$$U(h; z) = b(h) + \beta\, \mathbb{E}_{z'}\big[S^U(h; z') \,\big|\, z\big], \tag{3}$$

where $S^U(h; z)$ is the value of searching while unemployed for a worker of skill level h when the aggregate state is z, given by

$$S^U(h; z) = \max_{s,w}\Big\{U(h; z) + p\big(\theta(h, s, w; z)\big)\Big(\mathbb{E}_y\Big[W(h, s, w, y; z)\Big] - U(h; z)\Big)\Big\}. \tag{4}$$

Employed worker

Denote by $x = (h, s, w, y; z)$ the state of a worker with human capital h employed in sector s, with wage w, and idiosyncratic productivity y when the aggregate productivity level is z. Their value function is $W(x)$. Over their wage w, the worker is subject to a tax rate $\tau^w(s)$ given by

$$\tau^w(s) = \begin{cases} \tau^w > 0 & \text{if } s = F \\ 0 & \text{if } s = I, \end{cases} \tag{5}$$

that is, there is income tax only in the formal sector.

An ongoing match is subject to endogenous separation by the firm. If the firm wants to keep the match, there is still an exogenous separation rate $\delta(s)$. Let $d(x)$ represent this consolidated separation rate, which we define more precisely later in this section. If the match ends due to the firm choice or by exogenous reasons, the worker is entitled to receive benefits $B(s) \times w$, where the rate $B(s)$ is given by

$$B(s) = \begin{cases} B > 0 & \text{if } s = F \\ 0 & \text{if } s = I. \end{cases} \tag{6}$$

This captures the fact that only formal jobs are associated with (temporary) unemployment insurance, which we model as a one-time payment. A laid-off worker can search immediately as an unemployed worker.

Let $x' = (h, s, w, y; z')$. The value of an ongoing match for the worker with state x is

$$W(x) \ = \ \big(1 - \tau^w(s)\big)w \ + \ \beta\mathbb{E}_{z'}\left[W(x') + d(x')\Big(B(s)w \ + \ S^U(h; z') - W(x')\Big) \,\middle|\, z\right]. \tag{7}$$

Note that the only state variable that changes between periods for an employed worker is the aggregate productivity level z. This change might trigger a separation decision by the firm.

Value of a vacancy

A vacancy posted by a firm in the submarket $\eta = (h, s, w; z)$ costs κ_s to be maintained open and has an associated value of

$$V(\eta) = -\kappa_s + q\big(\theta(\eta)\big)\mathbb{E}_y\left[J(h, s, w, y; z)\right]. \tag{8}$$

The expected value over y is present because the idiosyncratic productivity y is drawn from G after the match is formed, yielding value $J(h, s, w, y; z)$ for the firm.

With free entry, it must be the case that

$$\kappa_s \geq q\big(\theta(\eta)\big)\mathbb{E}_y\left[J(h, s, w, y; z)\right] \tag{9}$$

and $\theta(\eta) \geq 0$ with complementary slackness. That means that (9) holds with equality ($V(\eta) = 0$) in every submarket where firms post vacancies (markets where $\theta(\eta) > 0$). Hence, the equilibrium condition for vacancy posting can be rewritten as

$$\theta(\eta) = q^{-1}\left(\frac{\kappa_s}{\mathbb{E}_y\left[J(h,s,w,y;z)\right]}\right) \quad \text{if } \theta(\eta) > 0. \tag{10}$$

Value of a match to the firm

When a match of type $x = (h,s,w,y;z)$ is maintained, the firm enjoys a profit flow $\pi(x)$ given by

$$\pi\big(x\big) = \big(1-\tau^p(s)\big)\Big(e^{A_s+z+y} \times h \; - \; \big(1+\tau^f(s)\big)w\Big) \tag{11}$$

The firm is subject to sector-specific payroll taxes and profit taxes at rates given by

$$\tau^f(s) = \begin{cases} \tau^f & \text{if } s = F \\ 0 & \text{if } s = I \end{cases} \tag{12}$$

and

$$\tau^p(s) = \begin{cases} \tau^p & \text{if } s = F \\ 0 & \text{if } s = I. \end{cases} \tag{13}$$

After the aggregate uncertainty is resolved, the firm can choose to terminate a no longer profitable match. The match is terminated with probability one if the continuation value for the firm is inferior to the cost of separation, $C(s)$, with

$$C(s) = \begin{cases} C > 0 & \text{if } s = F \\ 0 & \text{if } s = I. \end{cases} \tag{14}$$

Otherwise, the match ends accordingly to the exogenous separation rate $\delta(s)$ – which might differ by sector:

$$\delta(s) = \begin{cases} \delta_F \geq 0 & \text{if } s = F \\ \delta_I > 0 & \text{if } s = I. \end{cases} \tag{15}$$

The value for the firm of an ongoing match $x = (h,s,w,y;z)$ is thus given by

$$\begin{aligned} J\big(x\big) \;\; &= \; \pi\big(x\big) \; + \; \beta\mathbb{E}_{z'}\left[\max_d\Big\{d\big(-C(s)\big) \; + \; \big(1-d\big)J\big(x'\big)\Big\}\Big|\; z\right] \\ &\text{s.t.} \quad \delta(s) \leq d \leq 1. \end{aligned} \tag{16}$$

3.2 Distribution of workers across states

Let $\mu_z(a)$ denote the measure of employed workers in a match of a given type $a = (h,s,w,y)$ when the aggregate state is z and $u_z(h)$ be the measure of unemployed workers over state $(h;z)$, both at the production stage.

In a given instant, the inflow of people into unemployment is formed by the employed workers who were laid off and were unsuccessful in their search attempt:

$$\sum_a \Big[\mu_z(a) \times d(a;z)\big(1 - p\big(\theta(h, m^*; z')\big)\Big], \tag{17}$$

where $m^* = m^U(h; z')$.

On the other hand, the outflow from unemployment is composed of the individuals who were matched to a firm and drew high enough productivity to sustain the match in their submarket of choice:

$$\sum_h \Big(u_z(h) \times p\big(\theta(h, m^*; z')\big)\Big). \tag{18}$$

3.3 The government

The government has four sources of revenue:

i) Income tax at rate τ^w charged only of workers in formal matches:

$$\tau^w \times \sum_{h,w,y} \Big(w \times \mu_z(h, F, w, y)\Big)$$

ii) Payroll tax at rate τ^f charged only of firms in formal matches:

$$\tau^f \times \sum_{h,w,y} \Big(w \times \mu_z(h, F, w, y)\Big)$$

iii) Profit tax at rate τ^p charged only of firms in formal matches:

$$\tau^p \times \sum_{h,w,y} \Big[\Big(e^{A_s+z+y} \times h - \big(1 + \tau^f(s)\big)w\Big) \times \mu_z(h, F, w, y)\Big]$$

iv) Firing costs C charged of formal matches subject to separation:

$$C \times \sum_{h,w,y} \Big(\mu_z(h, F, w, y) \times d\big(h, F, w, y; z'\big)\Big),$$

where the timing should be noted: $\mu_z(x)$ is the measure of workers in matches x at the production stage when the aggregate state is z. Next period, the aggregate state is z', which leads to firing decisions ruled by $d(x; z')$. Hence, at tomorrow's firing stage, the measure of workers with state x is still $\mu_z(x)$.

On the expenditure side, the government provides unemployment insurance payments B to workers being exogenously separated from formal matches. The amount paid by the government in benefits (in the production stage next period) is

$$B \times \sum_{h,w,y} \Big(w \times \mu_z(h, F, w, y) \times d\big(h, F, w, y; z'\big) \Big)$$

Block recursivity requires that we relax the requirement of a balanced government at every period.

3.4 Equilibrium

Let $x = (h, s, w, y; z)$ represent a match state and $\eta = (h, s, w; z)$ represent a submarket. A recursive equilibrium in this economy is a set of individual policy functions for unemployed search $m^U(h; z)$, firing by the firms $d(x)$, market tightness functions $\theta(\eta)$, and distributions $u_z(h)$ and $\mu_z(h, s, w, y)$ for the unemployed and employed workers over their states, respectively, such that:

1. Agents' decision rules are optimal.
2. The labor market tightness satisfies the free entry condition given in Equation 10.
3. The distribution of workers across states is consistent with individual policy functions.

Appendix A derives the existence and uniqueness of the model's block recursive equilibrium (BRE) using the Banach Fixed-Point Theorem. The key implication of the BRE is that the equilibrium objects in conditions (1) and (2) depend on the aggregate state only through z and not through the distributions of workers, a complicated multidimensional object. This property allows the "block" of value functions and *prices* (the market tightness) to be solved independently of the aggregate state distribution.

Furthermore, given the absence of on-the-job search and worker-driven separations, our model affords a further simplification: the firm block (value functions J and V, separation policy d, and market tightness θ) can be solved in a first stage, independently of the worker's value functions. This two-stage decoupling, which is not a required feature of all BRE models, significantly simplifies our proof and computational solution.

4 Quantitative Illustration and Policy Experiments

In this section, we calibrate the model to match key qualitative features of the Brazilian labor market and conduct policy experiments. We start by defining the specific functional forms for the functions presented in the model and then discuss the calibration of the model's parameters. This quantitative exercise allows us to illustrate the role of the informal sector and of labor market regulation throughout the business cycle.

4.1 Parametrization

Table 4 summarizes the six functions in the model and the selected functional forms specified for each of them. Employed and unemployed workers enjoy linear utility from their income. The home production function is set as two discrete values, one for each human capital, $b_H > b_L$. Production is a function of the human capital of the worker, the productivity of the sector in which they work, the idiosyncratic productivity of the match, and the aggregate productivity level. Sector, idiosyncratic, and aggregate components jointly impact the productivity of the worker through an exponential term.

Table 4: Functional forms of the model

Function	Description	Functional Form	
$u(w)$	Utility function	Linear	$u(w) = w$
$b(h)$	Home production function	Discrete	$b(h) \in \{b_L, b_H\}$
$f(\cdot)$	Production function	Exponential	$f(h, s, y, z) = e^{A_s + y + z} \times h$
$M(u, v)$	Matching function	DRW Function	$M(u, v) = \frac{u \cdot v}{(u^\alpha + v^\alpha)^{\frac{1}{\alpha}}}$
$G(y)$	Distribution of y	Normal	$y \sim N(\mu_y, \sigma_y^2)$
$\Phi(z)$	Law of motion of z	AR(1)	$z_t = \rho_z z_{t-1} + \epsilon_t$, where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$

The constant returns to scale matching function probability follows den Haan, Ramey, and Watson (2000), and it is such that $p(\theta) = \theta\,(1 + \theta^\alpha)^{-\frac{1}{\alpha}}$. We assume the idiosyncratic productivity y is drawn from a normal distribution with mean μ_y and standard deviation σ_y. For the aggregate productivity level, we assume it follows a mean-zero AR(1), which we discretize using Rouwenhorst method as a five state grid: $Z = [z_1, z_2, z_3, z_4, z_5]$, where $z_1 < z_2 < 0$ are bad states (recessions), $z_3 = 0$ is a neutral state, and $0 < z_4 < z_5$ are good states.

The model's parameters are set in two groups. The first group consists of the externally calibrated parameters, whose values are drawn from institutional rules or calculated directly from the data (as detailed in Appendix B). This approach anchors the model to key real-world observations. They are presented in Table 5.

Table 5: Externally Calibrated Parameters

Symbol	Value	Description	Source / Rationale
h_L	1.0	Low-skilled human capital	Model normalization
A_I	0.0	Informal sector productivity	Model normalization
π_H	0.5415	Share of high-skilled workers	PME
β	0.99488	Discount factor	Set to match monthly real interest rate
α	0.2	Matching function elasticity	From Menzio and Shi (2010)
B	3.0	UI benefits multiplier	Statutory rules
τ^w	0.11	Income tax rate	Max. effective tax rate (Sindifisco Nacional, 2023)
τ^f	0.375	Payroll tax rate	Statutory values (Bosch & Esteban-Pretel, 2012; Ulyssea, 2018)
τ^p	0.0	Profit tax rate	Simplified out in this version

The second group is of the internally calibrated parameters. Their values are chosen jointly with the goal of making the model's simulated behavior match key moments from the data. This includes parameters for which no direct empirical counterpart exists or for which there is no available data. We search for a parameter vector that generates the qualitative cyclical patterns of the labor market, including an countercyclical informal sector more heavily populated by low-skilled workers, by providing a reasonable fit for key targeted moments, listed in Table 7. We use the full SMM objective function (detailed in Appendix B) as a diagnostic tool to guide this calibration. The resulting parameter values are presented in Table 6.

Table 6: Internally Calibrated Parameters

Parameter	Description	Value
b_L, b_H	Home Production	(0.05, 0.40)
h_H	Relative Human Capital	1.70
A_F	Formal Productivity	0.2624
κ_F, κ_I	Vacancy costs	(0.35, 0.10)
δ_F, δ_I	Exogenous separation rates	(0.006, 0.026)
C	Firing costs	4.00
$\bar{m}_w$	Minimum wage	3.00
μ_y, σ_y	Idiosyncratic shock parameters	(1.06, 0.08)
$\rho_z, \sigma_\varepsilon$	Aggregate shock parameters	(0.90, 0.25)

Given its directed search mechanism and absence of on-the-job search, this version of the model depends on the aggregate state variation to generate dispersion: it does not make sense to focus on a steady-state solution, as it would generate only two active markets, one for low-skilled and one for high-skilled workers, with each group entirely in the same sector. We can only obtain non-degenerated informality rates and transitions (that is, active formal and informal sectors for workers of the same skill level) through the business cycle variation. Although the model is capable of generating the broad main cyclical patterns of the Brazilian labor market, this small set of "states" prevents a perfect quantitative fit and leads to high volatility of both stocks and flows.

Table 7: Model Fit: Data vs. Model Moments

Moment	Data	Model
Key moments		
Unemployment rate, low-skilled	11.6671	15.2514
Unemployment rate, high-skilled	11.4529	8.3717
Informality rate, low-skilled	33.8790	21.1760
Corr(Unemployment, Activity)	-0.5755	-0.6158
Corr(Informality, Activity)	-0.3463	-0.1870
High- vs. low-skilled wage premium (log points)	0.5878	0.5294
Formal vs. informal wage premium (log points)	0.0751	0.4446
Other moments		
Informality rate, high-skilled	19.0439	0.3495
Formal job-finding rate (UF), low-skilled	8.2200	4.3613
Formal job-finding rate (UF), high-skilled	9.4375	7.7852
Informal job-finding rate (UI), low-skilled	12.6513	2.4596
Informal job-finding rate (UI), high-skilled	8.0309	0.6356
Formal separation rate (FU), low-skilled	0.8840	0.8221
Formal separation rate (FU), high-skilled	0.9511	0.6910
Informal separation rate (IU), low-skilled	3.0896	2.5142
Informal separation rate (IU), high-skilled	3.2004	12.9706
Minimum-to-average wage ratio (low-skilled formal)	0.6110	1.0406
Std. dev. unemployment rate	0.8972	2.6150
Std. dev. informality rate	0.6115	2.0518
Std. dev. formal separation rate	0.1029	1.4088
Std. dev. informal separation rate	0.4358	2.7906

4.2 Simulation and Policy Analysis

To measure the dynamic role of the informal sector during changing economic conditions, we simulate an economy with $N = 5,000$ individuals, respecting the distribution of skill types, for 650 periods.[6] We obtain randomly generated trajectories for the aggregate productivity shock, the draws of the idiosyncratic productivity levels, the exogenous separation shocks, and the realization of the matching probabilities. We identify recession periods as moments when the aggregate productivity level reaches negative values (z_1 and z_2), and "normal times" when it assumes the intermediary or positive levels.

The baseline model successfully reproduces the counter-cyclical behavior of the unem-

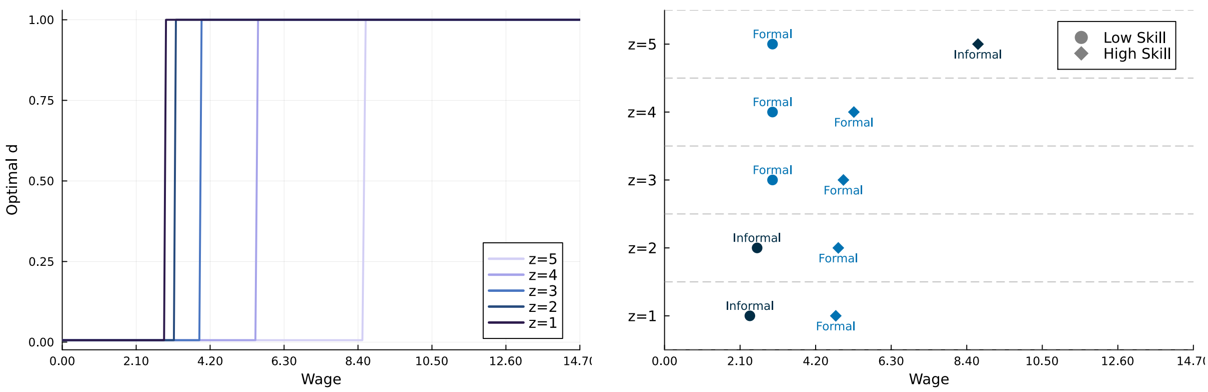
[6]More precisely, we simulate the model for 1,000 periods but drop the initial 350.

ployment and informality rates observed in the data. Recessions lead to an increase in the endogenous separations by firms, as previously profitable matches are no longer sustainable given the worsened aggregate state. This is illustrated in Panel (a) of Figure 2, which shows the endogenous separation threshold of a firm for different wages w and aggregate state levels z, in a match of type $(h = L, s = F, y = \mu_y)$. For an active match, the wage is fixed. When the aggregate state changes, the separation threshold shifts and the fixed wage may no longer be profitable, triggering the firm to terminate the job relation. The specific threshold level depends on the other match characteristics, but the general pattern is that, during downturns, the set of wages the firm is willing to pay is reduced. As shown in the figure, a shift from a high state ($z = 5$) to a low state ($z = 1$) causes the threshold wage to fall from approximately 8.4 to 3.0.

Relatedly, unemployed workers switch their choice of which submarket to search in towards the informal sector, as it presents higher matching probabilities during downturns. It's worth highlighting that such probabilities are not exogenous, but equilibrium objects resulting from the optimal decisions of workers and firms engaging in the search and match effort. Figure 2b reveals that this counter-cyclical switching is a key margin of adjustment, but only for low-skilled workers (circles). While they search in the formal sector during booms (states $z = 3, 4, 5$), they switch to searching in the informal sector during recessions (states $z = 1, 2$). This endogenous search decision is thus a primary driver of the counter-cyclical flows into informality. High-skilled workers (diamonds), on the other hand, find it optimal to search for a formal job in all but the highest aggregate state, in which the informal sector becomes preferred. Its lack of compliance with regulatory costs supports the offer of higher wages during a boom.

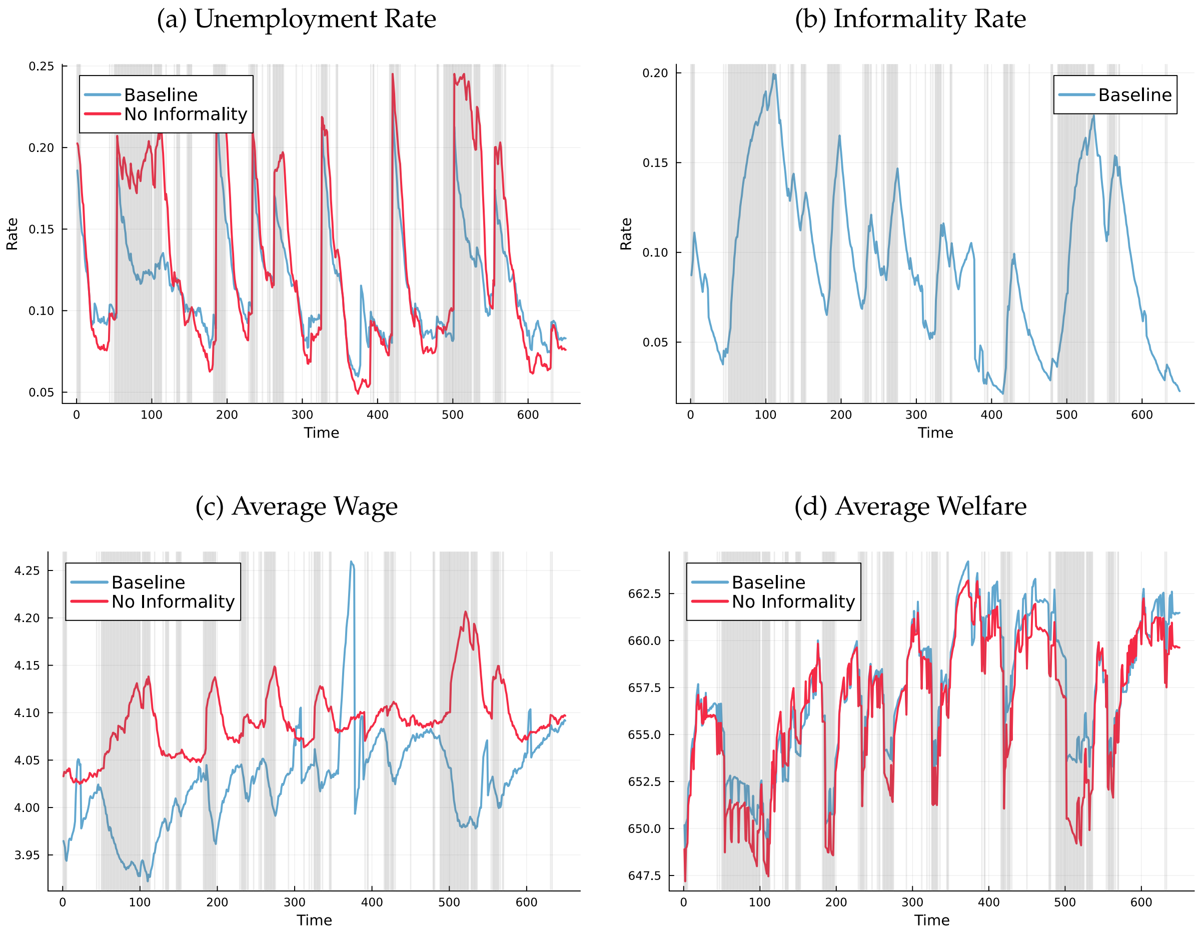
Figure 2: Unemployed Workers and Firms' Policy Functions

(a) Endogenous Separation Decisions (b) Unemployed Search Choices



Panels (a) and (b) of Figure 3 show the trajectories of the unemployment and the informality rates for the baseline scenario as blue lines. To highlight the role of the informal sector, we compare the baseline results to an alternative version of the model where the informal sector is shut down. More specifically, we prohibit matches in the informal sector by setting their probabilities to zero. The counterfactual trajectories for this alternative setting in Figure 3 are the red lines. In a world without informality, the unemployment rate would be systematically higher during recessions. As aggregate conditions deteriorate, the regulation in the formal sector becomes too burdensome for less productive matches, as they may no longer produce enough to sustain the costs associated with payroll taxes and the minimum wage. During expansions, however, the unemployment rate is slightly lower without informality, as the formal sector has lower separation rates and no"inherited" informal matches from bad times exist in this economy.

Figure 3: Labor Market Outcomes: Baseline vs No Informality



Notes: The figure shows the evolution of key labor market outcomes comparing the baseline model (with informality) to a counterfactual model without informality. Gray shaded areas indicate recession periods.

Panel (c) shows the average wage among employed workers in the two economies. The world without an informal sector generally has higher wages: the formal sector offers better pay, as it is more productive and bound to pay at least the minimum wage. However, this higher average wage is also due to composition effects among the employed population: workers who would be receiving less in the informal sector are, instead, unemployed. This phenomenon is so strong that it leads to contrary cyclical behavior of the average wage between the two economies. In the baseline model, labor income falls during recessions and increases in normal times. The opposite is true for the setting without informality, because when less productive matches end during recessions, with the formal sector inaccessible, workers become unemployed. Only the most productive individuals and matches, associated with the higher wages, remain active. When economic conditions improve, less productive workers return to the workforce, bringing down the average wage. Compared to the baseline scenario, the overall welfare effect is negative, with the largest gaps occurring during recessions (see Panel d). This points to the cyclical buffer role played by the informal sector.

Table 8 compares the outcomes of the baseline simulation with the counterfactual without informality, broken down by skill level. Given the limited combination of states in the model, although the segmentation of the informal sector is not complete, it is mostly comprised of low-skilled workers. For this group, the presence of informality substantially reduces unemployment rates, an effect that stems primarily from recession periods. While these workers earn lower average wages in the baseline model, their overall welfare is higher. This suggests that the increased employment opportunities and flexibility provided by the informal sector outweigh the associated wage penalty.

In contrast, high-skilled workers operate almost exclusively in the formal sector, so they experience very little difference in outcomes between the two scenarios. This contrast in effects across skill levels emphasizes that informality primarily serves as a buffer for vulnerable workers, particularly during economic downturns. These findings illustrate the complexity of policy decisions regarding informal labor markets, as measures to reduce informality without proper adjustment of social safety nets could disproportionately affect the most vulnerable workers.

Table 8: Labor Market Outcomes by Skill Level and Economic Conditions

	Baseline (A)		No Informality (B)		Difference (B-A)	
	Recession	Normal	Recession	Normal	Recession	Normal
Low-Skilled Workers:						
Unemployment	0.177	0.139	0.253	0.141	0.076	0.002
Welfare	454.630	460.322	451.240	459.285	-3.390	-1.037
Wage	2.850	2.901	2.983	2.991	0.133	0.090
High-Skilled Workers:						
Unemployment	0.105	0.072	0.106	0.070	0.001	-0.003
Welfare	823.061	827.150	822.974	827.217	-0.086	0.067
Wage	4.894	4.940	4.901	4.933	0.007	-0.007

Notes: This table shows average labor market outcomes by worker skill level and economic conditions. The last two columns show the difference between the baseline model with informality (A) and the model without informality (B).

The model provides a valuable framework for analyzing policy interventions over the instruments governing the formal labor market regulation. To illustrate this point, we observe the labor market outcomes of conducting counterfactual exercises on a few key policy dimensions: (1) the minimum wage policy ($\bar{w}_m$), comparing scenarios with 10% lower and 10% higher wage floors relative to the baseline; (2) firing costs (C), comparing scenarios where it is increased or decreased by 25%; and (3) payroll taxes (τ^f), analyzing the effects of 10% reductions and increases in the formal sector tax rate. For each counterfactual, we simulate the economy using the same sequence of aggregate and idiosyncratic shocks as in the baseline model, allowing us to isolate the pure policy effects. In Table 9, we show how each counterfactual policy instrument affects unemployment, informality rates, and welfare across worker skill levels, providing insights into the aggregate and distributional impacts of labor market regulations.

The minimum wage counterfactual exercises reveal heterogeneity in policy impacts across worker types. In this calibrated setting, a 10% reduction in the minimum wage generates improvements for low-skilled workers: both unemployment and informality fall, generating a welfare increase relative to the baseline scenario. Conversely, raising the minimum wage by 10% increases unemployment and informality while significantly reducing welfare. These changes are driven by switching in the sector choices of unemployed low-skilled workers when searching for a job. High-skilled workers remain unaffected by minimum wage changes, suggesting they operate well above this threshold.

Table 9: Policy Counterfactuals: Impact on Labor Market Outcomes

	Low-Skilled			High-Skilled		
Policy	Unemp.	Inform.	Welfare	Unemp.	Inform.	Welfare
Baseline ($\bar{w}_m = 3.0, C = 4.0, \tau^f = 0.375$)	0.153	0.212	458.325	0.084	0.003	825.716
Minimum wage						
Lower: $\bar{w}_m = 2.70$	-0.064	-0.212	7.811	0.000	0.000	0.000
Higher: $\bar{w}_m = 3.30$	0.092	0.782	-35.712	0.000	0.000	0.000
Firing cost						
Lower: $C = 3.00$	-0.001	-0.001	0.546	-0.001	-0.000	0.362
Higher: $C = 5.00$	0.000	0.001	1.828	0.001	0.000	-0.381
Payroll tax						
Lower: $\tau^f = 0.34$	-0.033	-0.155	20.194	0.000	0.000	23.233
Higher: $\tau^f = 0.41$	0.045	0.374	-21.527	-0.003	-0.000	-23.047

Notes: First row shows baseline levels. All other entries show differences relative to baseline. Results correspond to the entire simulation period, including both normal and recession periods.

We find that changes in firing costs (C) produce very small aggregate and welfare effects, as the unemployed worker's optimal search decisions are unaltered by a 25% increase or decrease in C. The modest welfare variations that do occur stem indirectly from the firm's side. The change in C slightly alters firms' value functions, which marginally adjusts equilibrium job-finding probabilities, leading to minor shifts in unemployment and informality.

As for payroll tax rates, changes in policy induce variation in both unemployment and informality for low-skilled workers. This is driven by changes in their optimal choice of sector to search in: a lower (higher) payroll tax induces low-skilled workers to switch their search towards (away from) the formal sector in some aggregate states. In contrast, the employment status of high-skilled workers remains unchanged in both experiments. However, the variation in the level of the tax rate also induces changes in the optimal wage choice for both skill groups. This explains the welfare impact observed for high-skilled workers.[7]

5 Concluding Remarks

We have proposed a model capable of incorporating aggregate uncertainty into a search and matching framework with informality in order to understand its dynamic role along the business cycle. In a quantitative illustration, we have shown that the model reproduces the main cyclical patterns of the informality and the unemployment rates observed in the data, with distributional implications as workers of different skill levels are affected differently. The presence of an informal sector provides an additional margin of adjustment during downturns, particularly for low-skilled workers, reducing unemployment and improving welfare despite lower wages. Policy experiments highlight the trade-offs between reducing informality and increasing unemployment when altering formal sector regulations, emphasizing the need to consider both distributional effects and cyclical dynamics in labor market policy design.

In terms of next steps, the current version of the model underperforms in generating a level of wage and sectoral match dispersion compatible with the data. In particular, the almost complete segmentation of the informal sector by skill is at odds with the data. This limitation stems from the small number of distinct searching worker groups – only ten combinations result from two skill types across five aggregate states –, which restricts the variety of matches in the economy and, in turn, limits the model's ability to match its full set of target moments. To better capture this heterogeneity and enable a proper estimation via a two-step Simulated Method of Moments (SMM), natural extensions include adding more granular human capital levels or incorporating an on-the-job search (OJS) mechanism. These modifications would not violate the model's block recursive structure. However, OJS would require adjusting how searching submarkets are defined, necessitating a move from the straightforward wage-posting setting to one with a lifetime utility index.

Finally, this framework can be of particular novelty in the analysis of the timing of labor market policy interventions. Are there persistent aggregate or distributional impacts of changing regulations during a specific phase of the business cycle rather than at another moment?

[7]Welfare analysis should be taken cautiously as this is not a balanced-budget exercise.

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Appendix

A Existence and Uniqueness of the Block Recursive Equilibrium

This section provides the proof for the existence of a unique Block Recursive Equilibrium (BRE) for the model. The proof leverages the Banach Fixed-Point Theorem in a two-stage process, exploiting the decoupling between firm values/market tightness and worker values enabled by the absence of on-the-job search.

A.1 Model Setup Summary

We repeat here the main components of the model necessary for the proof. The states of the model are:

- Skill level $h \in \mathcal{H} = \{L, H\}$.
- Sector $s \in \mathcal{S} = \{F, I\}$.
- Fixed-wage contracts with $w \in [\underline{w}, \overline{w}]$.
- Idiosyncratic match productivity $y \in [\underline{y}, \overline{y}]$ drawn once from continuous CDF $G(y)$.
- Aggregate productivity $z \in Z = \{z_1, \dots, z_{N_z}\}$, a finite set.

Let $x = (h, s, w, y; z)$ be the state of an existing match, $x' = (h, s, w, y; z')$ be the state tomorrow and $\eta = (h, s, w; z)$ be a submarket/vacancy type.

Value of an existing match to the firm Firms enjoy a flow profit

$$\pi(x) = \left(1 - \tau^p(s)\right)\left(e^{A_s+z+y} \times h - \left(1 + \tau^f(s)\right)w\right).$$

The firm chooses whether to fire or continue the match, accounting for exogenous separation cost $C(s)$:

$$J(x) = \pi(x) + \beta \mathbb{E}_{z'}\left[\max\left\{-C(s), \left(1 - \delta(s)\right)J(x') - \delta(s)C(s)\right\} \middle| z\right] \tag{A1}$$

Given the fixed point J^* satisfying Eq. (A1), define the optimal total separation probability $d^*(x')$ for a match in state x':

$$d^*(x') = \begin{cases} 1 & \text{if } -C(s) > J^*(x') \\ \delta(s) & \text{otherwise.} \end{cases} \tag{A2}$$

This $d^*(x')$ is a well-defined function determined solely by J^*.

Value of posting a vacancy Let $V(\eta)$ be the expected net value of posting a vacancy in submarket η. The free entry condition requires $V(\eta) \leq 0$ for all η. In any active submarket where $\theta(\eta) > 0$, it must be that $V(\eta) = 0$. This implies:

$$\kappa_s = q\big(\theta(\eta)\big)\mathbb{E}_y\left[J(h, s, w, y; z)\right]$$

Solving for the equilibrium tightness $\theta^*(\eta)$ gives:

$$\theta^*(\eta) = \begin{cases} q^{-1}\left(\frac{\kappa_s}{\mathbb{E}_y[J^*(h,s,w,y;z)]}\right) & \text{if } \mathbb{E}_y\left[J^*(h, s, w, y; z)\right] > \kappa_s \\ 0 & \text{if } \mathbb{E}_y\left[J^*(h, s, w, y; z)\right] \leq \kappa_s, \end{cases} \tag{A3}$$

where J^* is the equilibrium value function.

Value of an unemployed worker

$$U(h, z) = b(h) + \beta\mathbb{E}_{z'}\left[S^U(h, z')\middle|z\right], \tag{A4}$$

where $S^U(h, z)$ is the value of searching,

$$S^U(h, z) = U(h, z) + M(h, z), \tag{A5}$$

and $M(h, z)$ is the maximum expected gain from search,

$$M(h, z) = \max_{(s,w)} p\big(\theta^*(h, s, w; z)\big)\Big(\mathbb{E}_y\left[W(h, s, w, y, z)\right] - U(h, z)\Big). \tag{A6}$$

Value of an employed worker

$$W(x) = (1 - \tau^w(s))w + \beta\mathbb{E}_{z'}\Big[W(x') + d^*(x')\Big(B(s)w + S^U(h, z') - W(x')\Big)\Big|\, z\Big]. \tag{A7}$$

A.2 Assumptions

Assumption 1 (DRW Matching Function and Productivity Distribution)**.**

(i) The matching function takes the form:

$$M(u, v) = \frac{uv}{(u^\alpha + v^\alpha)^{1/\alpha}}$$

for some $\alpha > 0$, which implies:

$$p(\theta) = \frac{\theta}{(1 + \theta^\alpha)^{1/\alpha}}$$
$$q(\theta) = \frac{1}{(1 + \theta^\alpha)^{1/\alpha}}$$

(ii) The productivity distribution $G : [\underline{y}, \overline{y}] \to [0,1]$ is continuous with full support on $[\underline{y}, \overline{y}]$.

Assumption 2 (Parameter Restrictions)**.** *All tax rates $\tau^w, \tau^f, \tau^p \in [0,1)$, UI benefit rate $B \geq 0$, firing cost $C \geq 0$, and separation rates $\delta_F, \delta_I \in [0,1)$ are finite. The discount factor is $\beta \in (0,1)$. Home production $b(h)$ is bounded for all $h \in \mathcal{H}$.*

A.3 Proof of Existence and Uniqueness

The proof proceeds in two stages, using the Banach Fixed-Point Theorem.

A.3.1 Stage 1: Existence and Uniqueness of J^*

Proposition 1. *Under Assumptions 1 and 2, there exists a unique, bounded, and continuous firm value function $J^*(h, s, w, y, z)$ satisfying Eq. (A1).*

Proof. Define the operator T_J on the space $\mathcal{J}$ of bounded, continuous functions $J : \mathcal{H} \times \mathcal{S} \times [\underline{w}, \overline{w}] \times [\underline{y}, \overline{y}] \times Z \to \mathbb{R}$.

$$(T_J J_{old})(x) = \pi(x) + \beta \mathbb{E}_{z'} \left[\max\{-C(s), (1 - \delta(s)) J_{old}(x') - \delta(s) C(s)\} \middle| z \right]$$

1. **State Space Completeness:** The state space is $\mathcal{X} = \mathcal{H} \times \mathcal{S} \times [\underline{w}, \overline{w}] \times [\underline{y}, \overline{y}] \times Z$. Since $\mathcal{H} = \{L, H\}$ and $\mathcal{S} = \{F, I\}$ are finite, $[\underline{w}, \overline{w}]$ and $[\underline{y}, \overline{y}]$ are compact intervals, and Z is finite, $\mathcal{X}$ is compact. The space $\mathcal{J}$ of bounded continuous functions on $\mathcal{X}$ equipped with the supremum norm $\|J\| = \sup_{x \in \mathcal{X}} |J(x)|$ is thus a complete metric space.

2. **T_J maps $\mathcal{J}$ to $\mathcal{J}$:**

   - *Boundedness:* $\pi(x)$ is bounded on the compact domain $\mathcal{X}$. $C(s)$ is bounded by Assumption 2. If J_{old} is bounded, the term inside the max is bounded, and the expectation (a finite sum) is bounded. Thus $T_J J_{old}$ is bounded.
   - *Continuity:* $\pi(x)$ is continuous in all arguments. J_{old} is continuous by assumption. The term $(1 - \delta(s)) J_{old}(x') - \delta(s) C(s)$ is continuous in all state variables. The $\max\{A, B\}$ of two continuous functions is continuous. The expectation $\mathbb{E}_{z'}[\cdot] = \sum_{z' \in Z} \text{prob}(z'|z) \cdot (\cdot)$ is a finite sum of continuous functions weighted by transition probabilities, hence continuous in the current state variables (h, s, w, y, z). Therefore $T_J J_{old}$ is continuous.

3. **T_J is a Contraction:** We verify Blackwell's sufficient conditions.

   - *Monotonicity:* If $J_A \geq J_B$ pointwise, then

     $$(1 - \delta(s)) J_A(x') - \delta(s) C \geq (1 - \delta(s)) J_B(x') - \delta(s) C$$

     for all x'. The max operator preserves monotonicity. Since $\beta \geq 0$ and transition probabilities are non-negative, $\mathbb{E}_{z'}[\cdot]$ preserves monotonicity. Thus $T_J(J_A) \geq T_J(J_B)$.

- *Discounting:* For $a > 0$ and any function J,

$$\begin{aligned} T_J(J+a) &= \pi(x) + \beta \mathbb{E}_{z'} \Big[\max \big\{ -C(s), (1-\delta(s))(J(x')+a) - \delta(s)C(s) \big\} \Big| z \Big] \\ &= \pi(x) + \beta \mathbb{E}_{z'} \Big[\max \big\{ -C(s), (1-\delta(s))J(x') - \delta(s)C(s) + (1-\delta(s))a \big\} \Big| z \Big] \\ &\leq \pi(x) + \beta \mathbb{E}_{z'} \Big[\max \big\{ -C(s), (1-\delta(s))J(x') - \delta(s)C(s) \big\} + (1-\delta(s))a \Big| z \Big] \\ &= T_J(J) + \beta(1-\delta(s))a \end{aligned}$$

  where the inequality follows from the general property that $\max\{A, B+c\} \leq \max\{A, B\} + c$ for $c \geq 0$.
  Let $\underline{\delta} = \min_{s \in \mathcal{S}}\{\delta(s)\} \geq 0$. Then $(1-\delta(s)) \leq (1-\underline{\delta})$ for all s, so:

$$T_J(J+a) \leq T_J(J) + \beta(1-\underline{\delta})a$$

  The modulus of contraction is $\gamma_J = \beta(1-\underline{\delta})$. Since $\beta \in (0,1)$ (Assumption 2) and $\underline{\delta} \in [0,1)$, we have $\gamma_J < 1$.

4. **Conclusion:** By Blackwell's Sufficient Conditions, T_J is a contraction mapping. By the Banach Fixed-Point Theorem, T_J has a unique fixed point $J^* \in \mathcal{J}$, which is bounded and continuous.

□

Corollary 1 (Continuity of θ^* with DRW Matching)**.** *Assume the matching function takes the form $M(u,v) = \frac{uv}{(u^\alpha + v^\alpha)^{1/\alpha}}$ for $\alpha > 0$. Then the equilibrium market tightness function $\theta^*(\eta)$ defined by Eq. (A3) is continuous in $\eta = (h,s,w;z)$.*

Proof. With the DRW matching function, we have:

$$q(\theta) = \frac{1}{(1+\theta^\alpha)^{1/\alpha}}$$

which is continuous and strictly decreasing with $q(0) = 1$ and $\lim_{\theta \to \infty} q(\theta) = 0$.

Continuity of the expected firm value: We first establish that $\mathbb{E}_y\left[J^*(h,s,w,y;z)\right]$ is continuous in $\eta = (h,s,w;z)$. Let $\{\eta_n\}$ be any sequence with $\eta_n \to \eta$ and write $J^*(\eta_n, y) = J^*(h_n, s_n, w_n, y; z_n)$. We need to show:

$$\lim_{n \to \infty} \mathbb{E}_y\left[J^*(\eta_n, y)\right] = \mathbb{E}_y\left[J^*(\eta, y)\right]$$

.

We apply the Dominated Convergence Theorem, which requires three conditions:

1. **Pointwise convergence:** For each fixed $y \in [\underline{y}, \overline{y}]$, we have $J^*(\eta_n, y) \to J^*(\eta, y)$ as $n \to \infty$.

Justification: Since J^* is continuous in all its arguments (Proposition 1), we have

$$J^*(h_n, s_n, w_n, y; z_n) \to J^*(h, s, w, y; z)$$

for any $y \in [\underline{y}, \overline{y}]$.

2. **Integrable dominating function:** There exists an integrable function $g(y)$ such that $|J^*(\eta_n, y)| \leq g(y)$ for all n and all $y \in [\underline{y}, \overline{y}]$.

   Justification: From Proposition 1, the function J^* is bounded on the compact state space. Let $M = \sup_x |J^*(x)| < \infty$. Then:

$$|J^*(\eta_n, y)| \leq M \text{ for all } n, y$$

   We can take $g(y) = M$, which is integrable: $\int_{\underline{y}}^{\overline{y}} g(y)\, dG(y) = M < \infty$.

3. **Integration with respect to a probability measure:** The distribution G defines a probability measure on $[\underline{y}, \overline{y}]$.

   This is satisfied by construction.

By the Dominated Convergence Theorem:

$$\begin{aligned}
\lim_{n\to\infty} \mathbb{E}_y[J^*(\eta_n, y)] &= \lim_{n\to\infty} \int_{\underline{y}}^{\overline{y}} J^*(\eta_n, y)\, dG(y) \\
&= \int_{\underline{y}}^{\overline{y}} \lim_{n\to\infty} J^*(\eta_n, y)\, dG(y) \quad \text{(interchanging limit and integral)} \\
&= \int_{\underline{y}}^{\overline{y}} J^*(\eta, y)\, dG(y) \\
&= \mathbb{E}_y[J^*(\eta, y)]
\end{aligned}$$

Since this holds for any sequence $\eta_n \to \eta$, the function $\mathbb{E}_y[J^*(\eta, y)]$ is continuous in η by the sequential characterization of continuity.

Interior of active region: Where $\mathbb{E}_y[J^*(\eta, y)] > \kappa_s$, the inverse function

$$q^{-1}(q) = \left(q^{-\alpha} - 1\right)^{1/\alpha}$$

is continuous (as a composition of continuous functions: power, subtraction, and root). Therefore, $\theta^*(\eta) = q^{-1}\left(\frac{\kappa_s}{\mathbb{E}_y[J^*(\eta,y)]}\right)$ is continuous as a composition of:

- The continuous function $\eta \mapsto \mathbb{E}_y[J^*(\eta, y)]$ (established above).
- The continuous function $r \mapsto \kappa_s / r$ (on $r > \kappa_s$).
- The continuous function q^{-1}.

At the boundary: Where $\mathbb{E}_y[J^*(\eta, y)] = \kappa_s$, as $\eta \to \eta_0$ with $\mathbb{E}_y[J^*(\eta, y)] \to \kappa_s^+$, we have:

$$\frac{\kappa_s}{\mathbb{E}_y[J^*(\eta, y)]} \to \frac{\kappa_s}{\kappa_s} = 1$$

.

By continuity of q^{-1} and the explicit formula:

$$\lim_{\eta \to \eta_0} \theta^*(\eta) = \lim_{q \to 1^-} q^{-1}(q) = \lim_{q \to 1^-} \left(q^{-\alpha} - 1\right)^{1/\alpha} = (1^{-\alpha} - 1)^{1/\alpha} = 0^{1/\alpha} = 0 = \theta^*(\eta_0)$$

Therefore, $\theta^*(\eta)$ is continuous everywhere. □

A.3.2 Stage 2: Existence and Uniqueness of (U^*, W^*)

Given the unique J^*, we derive V^*, θ^*, and the total separation probability function $d^*(x')$ (from Eq. A2) as fixed, bounded functions determined by J^*. Since J^* is continuous (Proposition 1), the function $d^*(x')$ is Borel measurable (continuous functions are measurable, and indicator functions of measurable sets are measurable). Furthermore, θ^* is continuous (Corollary 1).

Proposition 2. *Under Assumptions 1 and 2, given J^*, there exists a unique pair of bounded and measurable worker value functions (U^*, W^*) satisfying Eqs. (A4), (A5), (A7).*

Proof. Define the operator T_W on the space $\mathcal{W}$ of pairs of bounded, Borel measurable functions $\mathbf{V}_{old} = (U_{old}, W_{old})$ defined on their respective state spaces. The space $\mathcal{W}$ is equipped with the norm $\|\mathbf{V}\| = \max(\|U\|_\infty, \|W\|_\infty)$ where $\|\cdot\|_\infty$ denotes the supremum norm.

$$T_W(\mathbf{V}_{old}) = (U_{new}, W_{new}),$$

where

$$W_{new}(x) = \left(1 - \tau^w(s)\right)w + \beta \mathbb{E}_{z'}\left[(1 - d^*(x'))W_{old}(x') + d^*(x')\left(B(s)w + S^U_{old}(h, z')\right)\Big| z\right]$$
$$U_{new}(h, z) = b(h) + \beta \mathbb{E}_{z'}\left[S^U_{old}(h, z')\Big| z\right],$$

and $S^U_{old}(h, z) = U_{old}(h, z) + M_{old}(h, z)$, with M_{old} defined by Eq. (A6) using $\mathbf{V}_{old}$.

1. **State Space Completeness:** The state spaces for U and W are compact (finite sets for discrete components, compact intervals for continuous components). The space $\mathcal{W}$ of bounded, Borel measurable functions equipped with the supremum norm is thus a complete metric space.

2. **T_W maps $\mathcal{W}$ to $\mathcal{W}$:**

   - *Boundedness:*
     - For W_{new}: The flow wage $(1 - \tau^w(s))w$ is bounded on the compact domain. The expectations involve bounded functions W_{old}, U_{old} (by assumption), bounded $d^* \in [\underline{\delta}, 1]$, bounded $B(s)$, and bounded w. Therefore W_{new} is bounded.

- For U_{new}: The flow utility $b(h)$ is bounded by Assumption 2. The expectation of the bounded function S^U_{old} is bounded. Therefore U_{new} is bounded.

- *Measurability:*
  - The objective function in $M_{old}(h,z)$ is measurable in (s,w) because it depends on measurable functions W_{old}, U_{old}, the continuous CDF G, the continuous function θ^* (Corollary 1), and the continuous function $p(\cdot)$. The maximum over a compact set of a measurable function yields a measurable function (Measurable Maximum Theorem). Thus $M_{old}(h,z)$ is measurable, and hence S^U_{old} is measurable.
  - W_{new} is measurable as a composition and expectation of measurable functions: W_{old} and S^U_{old} are measurable by assumption, d^* is measurable, flow wages are continuous (hence measurable), and expectations preserve measurability.
  - U_{new} is measurable as an expectation of the measurable function S^U_{old}.

3. T_W **is a Contraction:** We show $\|T_W(\mathbf{V}_A) - T_W(\mathbf{V}_B)\| \leq \beta \|\mathbf{V}_A - \mathbf{V}_B\|$.

- Let $\|\mathbf{V}_A - \mathbf{V}_B\| = d = \max(\|W_A - W_B\|_\infty, \|U_A - U_B\|_\infty)$.
- For any state (h,z), let $m_A = (s_A, w_A)$ be the optimal choice maximizing $M_A \equiv M(h,z;\mathbf{V}_A)$ and m_B be optimal for $\mathbf{V}_B$.
- **Step 1: Bound on $|S^U_A - S^U_B|$:**
  By optimality of m_A for $\mathbf{V}_A$:

$$M_A \geq p(\theta^*(h,m_B;z))\Big(\mathbb{E}_y\big[W_A(h,m_B,y,z)\big] - U_A(h,z)\Big)$$

  Therefore:

$$\begin{aligned} S^U_A - S^U_B &= (U_A - U_B) + (M_A - M_B) \\ &\geq (U_A - U_B) + p_B\Big(\mathbb{E}_y\big[W_A(\cdot)\big] - U_A\Big) - p_B\Big(\mathbb{E}_y\big[W_B(\cdot)\big] - U_B\Big) \\ &= \big(U_A - U_B\big)\big(1 - p_B\big) \;+\; p_B\,\mathbb{E}_y\Big[W_A(\cdot) - W_B(\cdot)\Big], \end{aligned}$$

  where $p_B = p(\theta^*(h,m_B;z))$.
  Since $U_A - U_B \geq -d$ and $W_A - W_B \geq -d$ pointwise, and $p_B \in [0,1]$:

$$S^U_A - S^U_B \geq (-d)(1-p_B) + p_B(-d) = -d$$

  By symmetry (starting from $S^U_B - S^U_A$ and using optimality of m_B):

$$S^U_B - S^U_A \geq -d$$

  Therefore $|S^U_A - S^U_B| \leq d$ for all (h,z), which implies $\|S^U_A - S^U_B\|_\infty \leq d$.

- **Step 2: Contraction for U:**

$$\begin{aligned}\left|U_A^{new}(h,z) - U_B^{new}(h,z)\right| &= \left| \beta \mathbb{E}_{z'}\left[S_A^U(h,z') - S_B^U(h,z') \mid z\right] \right| \\ &\leq \beta \mathbb{E}_{z'}\left[\left|S_A^U(h,z') - S_B^U(h,z')\right| \mid z\right] \\ &\leq \beta \mathbb{E}_{z'}\left[d \mid z\right] = \beta d\end{aligned}$$

- **Step 3: Contraction for W:**

$$\begin{aligned}&\left|W_A^{new}(x) \ - \ W_B^{new}(x)\right| \\ &\quad = \left| \beta \mathbb{E}_{z'}\left[(1-d^*(x'))\big(W_A(x') - W_B(x')\big) + d^*(x')\big(S_A^U(h,z') - S_B^U(h,z')\big) \mid z\right] \right| \\ &\quad \leq \beta \mathbb{E}_{z'}\left[\ (1-d^*(x'))\left|W_A(x') - W_B(x')\right| \ + \ d^*(x')\left|S_A^U(h,z') - S_B^U(h,z')\right| \mid z \ \right] \\ &\quad \leq \beta \mathbb{E}_{z'}\left[\ (1-d^*(x'))d + d^*(x')d \mid z \ \right] \\ &\quad = \beta d\end{aligned}$$

- Therefore:

$$\|T_W(\mathbf{V}_A) - T_W(\mathbf{V}_B)\| = \max(\|W_A^{new} - W_B^{new}\|_\infty, \|U_A^{new} - U_B^{new}\|_\infty) \leq \beta d,$$

  so T_W is a contraction with modulus $\beta < 1$.

4. **Conclusion:** By the Banach Fixed-Point Theorem, T_W has a unique fixed point $\mathbf{V}^* = (U^*, W^*) \in \mathcal{W}$, which is bounded and measurable.

□

A.4 Conclusion

The two-stage proof demonstrates the existence and uniqueness of the firm value function J^* (bounded and continuous) and the worker value functions (U^*, W^*) (bounded and measurable). These value functions, along with the derived firm vacancy value V^*, market tightness θ^* (continuous), and associated optimal policy functions (worker search choice $m^*(h,z)$ and firm firing rule defining d^*), constitute the unique Block Recursive Equilibrium for the specified economy.

Note on Block Recursivity

The equilibrium derived in this proof possesses the Block Recursive property. This is because the core equilibrium objects determining market interactions – namely the firm's value function for existing matches (J^*), the firm's expected value from posting a vacancy (V^*), the resulting market tightness (θ^*), and the firm's optimal firing rule (d^*) – are all determined in Stage 1 of the proof *independently* of the worker value functions (U^*, W^*) and, crucially, independently of the aggregate distribution of unemployed workers.

This decoupling occurs because the model assumes no on-the-job search, preventing the distribution of employed workers from affecting market tightness. Consequently, the applicant pool for any vacancy consists only of unemployed workers of a specific type. As search is directed, everyone looking for a job in a particular submarket is willing to accept the job if matched to a firm in that submarket.

Stage 2 then solves for the worker value functions taking the results from Stage 1 as given parameters. This structure allows the "block" of value functions and prices to be solved separately from the aggregate state distribution, fulfilling the definition of a BRE. This contrasts with models featuring on-the-job search, where the interdependence between firm values, worker search strategies, and the worker distribution typically necessitates different proof techniques.

B Data and Estimation Methodology

This section details how we use the data to discipline the model's parameters. It describes the externally calibrated parameters, the internally estimated parameters, the construction of the empirical target moments, and the plan for the Simulated Method of Moments (SMM) estimation procedure to be implemented in a future version of this paper.

B.1 List of Parameters

The model contains a total of 23 structural parameters. We divide these into two groups: (1) externally calibrated parameters, and (2) internally estimated parameters (via SMM).

The set of externally calibrated parameters has their values drawn from the literature, from institutional rules, or calculated directly from the data. This approach reduces the dimensionality of the SMM estimation and anchors the model to key real-world observations. The list of externally calibrated parameters is presented in Table 5, in the main text.

The core of the estimation is a vector P of 14 structural parameters, which will be jointly estimated in a SMM procedure. These parameters and their primary identifying moments are listed in Table B1.

Table B1: Internally Estimated Parameters

Symbol	Description	Primary Identifying Moment(s)
Long-run average moments		
b_L, b_H	Home Production	Average unemployment rates (by skill level)
h_H	Relative Human Capital	High-skilled vs. low-skilled wage premium
A_F, κ_F, κ_I	Productivity & Vacancy costs	Informality rates and job-finding rates (by skill), and the formal-informal wage premium
δ_F, δ_I	Separation rates	Average levels of job-separation rates (by skill)
$\bar{m}_w$	Minimum wage	Minimum wage ratio
μ_y, σ_y	Idiosyncratic shock parameters	Jointly identified with other parameters by the labor market stock and flow moments
Cyclical moments		
C	Firing costs	Primarily identified by the cyclical sensitivity of job-separation rates
$\rho_z, \sigma_\varepsilon$	Aggregate shock parameters	Cyclical moments (std. dev. and correlations)

B.1.1 Defining skill and skill-shares

We define high-skilled workers as those with at least a high school degree, and low-skilled workers as those with less than a high school degree. This binary classification aligns with the educational information available in the PME microdata.

To set the fixed skill shares π_H and $\pi_L = (1 - \pi_H)$ of the model, we first calculate the share of high-skilled workers in the labor force for each month, creating a time series $\pi_H(t)$. We then set π_H as the average of this series over the full sample period.

B.1.2 Policy Parameters

Tax rates reflect Brazilian institutional features. The income tax rate is set at 11% income based on maximum effective tax rates from Sindifisco Nacional (2023). Payroll taxes have a 37.5% rate following the calculations from Ulyssea (2018) and Bosch and Esteban-Pretel (2012), which combine direct taxes and social security contributions. The profit tax rate τ^p is set to zero as a simplification.

Unemployment insurance payments depend on the worker's past wage, how long they were employed before layoff, and the interval since last receiving UI. The system provides 3 to 5 months of benefits, and there are floors (minimum wage) and ceilings to the amount of the benefits. To simplify, we assume $B = 3$, the minimum duration of benefits, as the payment in the model implies receiving the full wage (and not just a fraction of it).

B.2 Empirical Target Moments

We construct a vector M_{data} of 21 target moments from the data. The calculation of these moments requires a multi-step preparation process to ensure consistency between the empirical data and the model's simplified structure.

B.2.1 The Vector of Target Moments

Table B2: Empirical Target Moments

Labor Market Stocks (Avg. of education-detrended series)		
1	Avg. unemployment rate, low-skilled (%)	11.6671
2	Avg. unemployment rate, high-skilled (%)	11.4529
3	Avg. informality rate, low-skilled (%)	33.8790
4	Avg. informality rate, high-skilled (%)	19.0439
Labor Market Flows (Avg. of education-detrended and adjusted series)		
5	Avg. formal job-finding rate (UF), low-skilled (%)	8.2200
6	Avg. formal job-finding rate (UF), high-skilled (%)	9.4375
7	Avg. informal job-finding rate (UI), low-skilled (%)	12.6513
8	Avg. informal job-finding rate (UI), high-skilled (%)	8.0309
9	Avg. formal job-separation rate (FU), low-skilled (%)	0.8840
10	Avg. formal job-separation rate (FU), high-skilled (%)	0.9511
11	Avg. informal job-separation rate (IU), low-skilled (%)	3.0896
12	Avg. informal job-separation rate (IU), high-skilled (%)	3.2004
Wage Structure		
13	Formal-informal wage premium (log points)	0.0751
14	High-skilled vs. low-skilled premium (log points)	0.5878
15	Minimum wage ratio	0.611
Cyclical Moments		
16	Std. dev. of unemployment rate, overall (%)	0.8972
17	Std. dev. of informality rate, overall (%)	0.6115
18	Std. dev. of formal job-separation rate (FU), overall (%)	0.1029
19	Std. dev. of informal job-separation rate (IU), overall (%)	0.4358
20	Correlation of unemployment rate and cycle component activity	-0.5755
21	Correlation of informality rate and cycle component activity	-0.3463

B.2.2 Data Preparation: Adjusting for Trends and Model Scope

The empirical labor market time series are affected by trends and data-model mismatches that must be accounted for.

1. Adjusting for Model Scope (Flows Adjustment) Our model is a closed 3-state system (Formal, Informal, Unemployed), but the data contains additional "exit" states corresponding to other employment conditions (self-employed, employers, public sector, and out of the labor force). To make the empirical data consistent with the model's structure, we must calculate flow probabilities conditional on remaining within the {F, I, U} system.

For any transition from an origin state $J \in \{F, I, U\}$ to a destination state $K \in \{F, I, U\}$, the adjusted probability is calculated as:

$$p_{JK}^{adj}(t) = \frac{\mathrm{N}_{J\to K}(t)}{N_J(t) - \mathrm{N}_{J\to X}(t)}, \tag{B1}$$

where $N_J(t)$ is the total stock in state J at time t, and $N_{J\to X}(t)$ is the number of individuals flowing from state J to any "exit" state $X \notin \{F, I, U\}$. All flow rates time series (Moments 5-12) are constructed using this adjustment method to the raw series.

Stocks (unemployment and informality rates) are calculated directly from the raw data, also considering the appropriate denominators (only private sector workers and unemployed).

2. Seasonal Adjustment First, for all raw monthly aggregated time series (which include the adjusted flow series from the step above), we apply a standard X-13 ARIMA-SEATS procedure to remove predictable seasonal fluctuations. We then apply a centered moving average to smooth high-frequency noise.

3. Educational Trend Removal Second, we observe a significant, slow-moving trend in the educational attainment of the workforce over our sample period. Our model, by design, assumes a fixed skill share (π_H). To make the data consistent with the model's structure, we must remove the effect of this compositional trend. We do this by:

1. Extracting the low-frequency trend from the high-skill share series, $\pi_H^{trend}(t)$, using an HP filter.

2. For each seasonally-adjusted labor market series $Y(t)$ of the previous step, we run the regression $Y(t) = \alpha + \beta\pi_H^{trend}(t) + \epsilon(t)$.

3. We construct the detrended series as $Y_{detrended}(t) = \text{mean}(Y) + \epsilon(t)$. This new series will share the same average as the original series, so this step does not affect the long-run average moments. It is critical, however, for the calculation of the cyclical moments, by removing the demographic trend component.

4. Cyclical Trend Removal Finally, to calculate the cyclical moments, we isolate the high-frequency business cycle component applying the Hodrick-Prescott (HP) filter to the fully cleaned $Y_{detrended}(t)$ series from the previous step. This step is also applied to the activity measure (smoothed log of seasonally adjusted IBC-Br), yielding its cyclical component.

B.2.3 Wage Premiums

The two wage premium moments are derived from Mincer regressions on the microdata panel, as described in the main text. To maintain consistency with the model scope, we include only private formal and informal wage workers. The results of these two regressions, from which Moments 13 and 14 are taken, are presented in Table B3.

Table B3: Wage Premium Regressions

	Formal Premium	Skill Premium
Formal sector	0.075***	0.205***
	(0.001)	(0.001)
Age	0.041***	0.058***
	(0.001)	(0.000)
Age squared	-0.000***	-0.001***
	(0.000)	(0.000)
Intercept		0.467***
		(0.003)
High-skilled		0.588***
		(0.001)
Female		-0.204***
		(0.001)
Individual fixed effects	Yes	–
Region dummies	–	Yes
N. Obs.	4,194,937	4,437,983
R^2	0.900	0.301
R^2 Adj.	0.873	0.301

Notes: * p < 0.1, ** p < 0.05, *** p < 0.01.

B.2.4 Minimum wage ratio

The minimum wage ratio moment (Moment 15) is calculated as the ratio between the average real monthly minimum wage over the sample period (set by law) and the average real monthly wage of low-skilled, full-time, private sector workers.

B.3 SMM Procedure

The SMM procedure consists in estimating the parameter vector P of Table B1 in a single, unified step by minimizing the distance between the data moments M_{data} (from Table B2) and the model-generated moments $M_{model}(P)$.

B.3.1 Weighting Matrix

To account for differences in the scale of the moments (e.g., rates vs. standard deviations) and to construct an efficient estimator, we use a weighting matrix. We employ a diagonal weighting matrix, $\tilde{W} = \text{diag}(W)$, where W is the variance-covariance matrix of the empirical moments estimated via a **block bootstrap** procedure with B repetitions:

1. We resample with replacement from the unique individual identifiers in our microdata panel.
2. For each bootstrap replication, we build a new panel dataset.
3. We re-run the entire empirical pipeline on this new panel: calculating the adjusted flow rates and stocks, seasonal adjustment, educational detrending, and the calculation of the full 21-moment vector.
4. W is computed as the covariance matrix of the $B \times 21$ matrix of resulting moments. $\tilde{W}$ contains only the diagonal elements of W, i.e., the variance of each moment.

B.3.2 Simulated Moments

For any given parameter vector P, finding the simulated moments $M_{model}(P)$ requires solving the full model and simulating it for N individuals over T periods. The simulated moments are calculated from this generated panel data using the equivalent recipes to their empirical counterparts:

- **Stock and Flow Moments** are the long-run averages of the simulated time series.
- **Wage Premiums** are calculated from the simulated panel using a composition-weighted method to handle potential empty cells.
- **Minimum Wage Moment**: We calculate the average real monthly wage of low-skilled, full-time, private workers from the simulation (W^L_{model}). The simulated moment is the ratio $\bar{w}_m / W^L_{model}$, where $\bar{w}_m$ is the parameter value being tested.
- **Cyclical Moments** are calculated by applying the same HP filter to the simulated aggregate series (e.g., log simulated GDP) and labor market series.

B.3.3 Minimization

The SMM procedure searches for the parameter vector P^* that minimizes the quadratic distance function:

$$P^* = \underset{P}{\operatorname{argmin}} \left[M_{data} - M_{model}(P)\right]' \tilde{W}^{-1} \left[M_{data} - M_{model}(P)\right].$$

This high-dimensional optimization will be performed in two stages: first, using a global search algorithm to explore the parameter space using endogenously built grids, and second, using a local search algorithm to refine the estimates in the region of the global optimum under fixed grids.